**Spring** 2010 **Exam 3** **Honors** 213

**April** 22

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*Name*

**Directions: Only write on one side of each page.**

## Do any (5) of the following

1. (20 points) Using any previous results, prove Proposition 4.7: Hilbert's Euclidean parallel postulate $\Longleftrightarrow$ if a line intersects one of two parallel lines, then it also intersects the other.

2. (20 points) Using any previous results, prove the **uniqueness** (but not existence) part of Proposition 4.3: Every segment has a unique midpoint.

3. (20 points) Using any results through Chapter 5 prove that Hilbert's Euclidean parallel property $\Longleftrightarrow$ Statement Ex5 where statement Ex5 is:

   Given lines $l$ and $m$ where $l \parallel m$, point $P$ is on $m$, $Q$ is the foot of the perpendicular from $P$ to line $l$, and $R$ is the foot of the perpendicular from $Q$ to line $m$. Then $\overleftrightarrow{PQ} = \overleftrightarrow{QR}$.

4. (20 points) Using any results through Chapter 4, prove the following: Hilbert's parallel property holds $\Longleftrightarrow$ if $k, m, l$ are distinct lines, $k$ is parallel to $m$, and $m$ is parallel to $l$, then $k$ is parallel to $l$.

5. (20 points) Using any results through Chapter 5, prove that Hilbert's parallel postulate implies Wallis' postulate. [Wallis' postulate is: Given any triangle $\triangle ABC$ and given any segment $DE$. There exists a triangle $\triangle DEF$ (having $DE$ as one of its sides) that is similar to $\triangle ABC$.]

6. (4 points each) Which of the following statements are correct? [You need not rewrite the statements when you answer.]

   (a) In hyperbolic geometry, if $\triangle ABC$ and $\triangle DEF$ are equilateral triangles and $\measuredangle A \cong \measuredangle D$, then the triangles are congruent.

   (b) In hyperbolic geometry, if $m$ contains a limiting parallel ray to $l$, then $l$ and $m$ have a common perpendicular.

   (c) In hyperbolic geometry, if $m$ does not contain a limiting parallel ray to $l$ and if $m$ and $l$ have no common perpendicular, then $m$ intersects $l$.

   (d) Every valid theorem of neutral geometry is also valid in hyperbolic geometry.

   (e) In hyperbolic geometry, there exists an angle and there exists a line that lies entirely within the interior of this angle.